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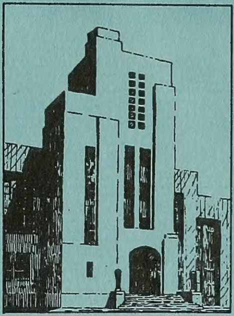
ON HYDRODYNAMIC MASSES

by

Georg P. Weinblum, D. Eng.



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NOTATION

a	Acceleration
B	Beam
F	$U/\sqrt{gL}$ Froude number
F_ω	$\omega\sqrt{B/g}$ Frequency parameter
g	Gravity acceleration
J	Mass moment of inertia
J_{ox}, J_{oy}, J_{oz}	Moment of the homogeneous underwater body with the density ρ
J_{xx}, J_{yy}, J_{zz}	Hydrodynamic moments of inertia
k	Inertia coefficient
k_x	m_x/m_o
k_{xx}	J_{xx}/J_{ox}
$\tilde{k}_x, \tilde{k}_y, \tilde{k}_z$	Added mass, free surface with gravity effects
$\mathring{k}_x, \mathring{k}_y, \mathring{k}_z$	Added mass, free surface without gravity effects
$\bar{k}_x, \bar{k}_y, \bar{k}_z$	Added mass, rigid wall
L	Length
m	Mass
m'	Apparent (virtual) mass, e.g., $m_x' = m_o + m_x$
m_x, m_y, m_z	Hydrodynamic (added) masses
$\tilde{m}_x, \tilde{m}_y, \tilde{m}_z$	Added mass, free surface with gravity effects
$\mathring{m}_x, \mathring{m}_y, \mathring{m}_z$	Added mass, free surface without gravity effects
$\bar{m}_x, \bar{m}_y, \bar{m}_z$	Added mass, rigid wall
O	Surface
p	Pressure
T	Kinetic energy
U	Speed of advance
u	Velocity
$\forall$	Volume
x, y, z	Coordinates

α	Angle of incidence
δ	Damping coefficient
ρ	Density of water
ϕ, Φ	Velocity potentials
ω	Circular frequency

ON HYDRODYNAMIC MASSES

by

Georg P. Weinblum, D.Eng.

The present synopsis is the English version of a paper in German presented on the sixtieth birthday of Professor Schnadel in Hamburg. Although it contains only a few original contributions by the author, it has been decided to publish the review as a TMB report in order that it may serve as an introduction to a subject which, despite its importance in many fields of hydrodynamics, has been somewhat neglected in the past.

In the present paper, we shall treat the problem of the hydrodynamic (added) masses of bodies which, like the ship, move through the free surface of the water. While for bodies in a medium extending infinitely in all directions the hydrodynamic inertia factors are defined purely geometrically except for the density of the medium, in our case dependencies on various quantities such as the Froude number, an acceleration ratio, a frequency parameter, etc., can appear. The nature of these dependencies is known only in a general way. It is the purpose of this study to give a review of the status of our knowledge in this field.

1. THE KELVIN FLOW FIELD

1.1 The study of the motion of a ship in water must, at the present, be carried out in various degrees of approximation. We shall assume here as an hypothesis that: the fluid is ideal, extends infinitely in all directions, and the flow in it is caused only by the motion of the body.

A "regular" (simply connected) body then produces an irrotational and acyclic velocity field which is characterized by a velocity potential ϕ and which, indeed, is also called the classical Kelvin field.[1] According to Kirchhoff, the kinetic energy of this field is given by a quadratic form in the velocity components of the body with the hydrodynamic (added) masses as the coefficients. The forces exerted by the fluid on the body can be defined by means of the hydrodynamic masses without performing the troublesome integration of the pressures over the body.

As usual we shall define the <u>apparent</u> or virtual mass in a given direction as the sum of the mass of the body m_o plus the hydrodynamic mass in that direction; we shall here set m_o equal to the mass of the displaced

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[1]References are listed on page 14.

fluid. This nonessential assumption, which practically speaking is generally justified in our case, simplifies the presentation.

As long as the fluid is assumed to extend infinitely in all directions, we shall avoid in definitions the expression "ship," although we are primarily interested in applications to the ship.

1.2 With regard to methods for calculating the hydrodynamic mass quantities or their coefficients, refer to, for example, the textbook of Lamb[1] and the dissertation of Wendel.[2] They are based on knowledge of the velocity potential ϕ, with whose help the expression for the kinetic energy or the components of the force and of the moment can be ascertained. For a "regular" body (without a hole) one obtains, as is well known, fifteen hydrodynamic inertia quantities, which number is reduced to six if the body has three planes of symmetry. Inasmuch as the hydrodynamic mass represents a tensor quantity, the notation becomes important. For example, a useful symbolism is obtained if one puts the kinetic energy

$$T = \frac{1}{2} \sum_{i=1}^{6} \sum_{j=1}^{6} m_{ij} u_i u_j \qquad [1]$$

where the components of the velocity of translation are $u_1 = u_x$, etc., and the components of the velocity of rotation are $u_4 = \omega_x$, etc. We cannot aspire to completeness here, however; on the contrary, we must frequently be satisfied with the implications only.

Therefore, we shall confine ourselves to body forms whose kinetic energy is determined by six hydrodynamic masses which we shall designate in the usual way by m_x, m_y, m_z for the translational motions and by J_{xx}, J_{yy}, and J_{zz} for the rotational motions. The corresponding <u>inertia coefficients</u> are then defined, although not entirely fortuitously, by

$$k_x = \frac{m_x}{m_o}, \qquad k_y = \frac{m_y}{m_o}, \text{ etc.,}$$

$$k_{xx} = \frac{J_{xx}}{J_{ox}}, \qquad k_{yy} = \frac{J_{yy}}{J_{oy}}, \text{ etc.}$$

Here, as previously indicated, m_o is the mass of the displaced fluid.

J_{ox}, J_{oy}, J_{oz} are the moments of inertia of the displaced fluid or of the <u>homogeneous</u> body having the density of water ρ. Naturally, J_{ox}, etc., differ in general from the moment of inertia of the body J_x, etc., although the mass of the body, as assumed here, is equal to the mass of the fluid.

For completeness, we note, in addition, that the symmetry of the

tensor m_{ij} in general vanishes in the presence of a free surface and that the conclusions of Kirchhoff concerning the influence of the symmetry of the body form are no longer applicable. Again, we must be satisfied with an indication only.

As originally indicated, the hydrodynamic inertia quantities in our case are functions of the body form and the density of the medium only.

The coefficients $k_x = m_x/m_o$, etc., are pure numerical values, which characterize a given geometrical configuration and which are roughly comparable to the fineness ratios in the various directions. These physical quantities are easily determined, and in this lies the great simplicity of the concept under consideration.

Despite the abstractness of this theory, it yields valuable practical results in cases where the basic hypotheses are satisfied. Above all, this is the case for accelerated motions of the body (ship) such as starting conditions, oscillations, vibrations, and impact phenomena.

However, even in calculating the moment M_y, which a body moving with a rectilinear uniform motion at an angle of attack α experiences and which can be calculated by the so-called Munk Formula

$$M_y = \rho(k_y - k_x)\forall U^2\alpha \qquad [2]$$

useful results are obtained if the circulation actually remains small as is the case for a "regular" body of revolution. For such a body, experiments give a "correction factor" of ~ 0.85. For ship models with a sharp stern, the author and others found a correction factor which naturally differed more from unity and in special cases amounted to ~ 0.60.

1.3 The explicit calculation of apparent masses is not simple even in the classical case. With regard to the two-dimensional problem, we refer to the treatment of K. Wendel.[2] The three-dimensional problem has been solved for the sphere and the ellipsoid; in addition, further results can be derived from a theorem of Munk,[3] according to which the apparent mass, for example, $m'_x = m_o + m_x$, of a body which is built up from a doublet distribution μ amounts to

$$m'_x = m_o + m_x = \frac{\rho}{U}\int_{\forall}\mu d\forall \qquad [3]$$

With this relation, the values of m_x and $m_y = m_z$ are given directly for a wide class of bodies of revolution. For the fundamental three-dimensional shapes which in simple cases can be produced by distributions of singularities in a plane (the so-called "Michell ship"), difficulty in constructing the body arises because no stream function exists. In fact, this problem as yet has not been treated at all, although it is solvable in principle.

Recently, several fundamental investigations of forms for which the sum $k_x + k_y + k_z$ is to be a minimum have been undertaken. The sphere and the circular cylinder play an important role here. However, these difficult treatments contribute nothing at the present to the solution of our problems.[4]

1.4 As is well known, existing results can be applied directly in ship theory as follows:[2] We image the submerged portion of the ship about the waterline and calculate the inertia coefficients of the double model in an infinite medium. At the same time, a strip method is applied and a correction for the end flow is made on the basis of the results for the ellipsoid. In many cases, one simply sets the values k_x, k_y, etc., equal to those of an "equivalent" ellipsoid.

The crude strip method frequently proves to be very good if the motion treated is in a vertical plane, therefore, if the determination of k_z and k_{yy} is the primary concern. As we shall see later, the values of Lockwood Taylor, Equation [12c], should then logically be used for motions in a horizontal plane. This, however, is not always done. Physically, the described procedure is naturally very unsatisfactory.

However, here we must emphasize a difference between the physical and the engineering considerations. From the latter standpoint, a special accuracy in determining the added masses is not necessary in many cases, so that one can be satisfied with approximate values. The problem of vibration, whose solution requires, among other things, very accurate knowledge of the hydrodynamic inertia values, constitutes an exception.

In the next section, we shall make some observations on what has been done and on what is being done at the present to arrive at a rational procedure.

2. FREE SURFACE

2.1 When one can no longer consider the fluid to extend infinitely in all directions, then it is obvious from physical considerations that the hydrodynamic masses depend upon the existing boundary conditions in addition to the body form. If, in particular, there is a free surface, then we must take the formation of waves into account. In order to differentiate the results in this case from our previous results, we shall provide the usual symbols with wavy lines, for example, $\tilde{m}_x$, $\tilde{k}_x$, etc.

First of all, consider a physical observation. It is clear that the kinetic energy can no longer be represented in the form of Kirchhoff because of the creation of waves. The kinetic energy, whose time rate appears as generated power or, in the case of oscillations, as the damping effect,

is constantly dissipated by progressive waves. A "reversible" part of the kinetic energy corresponds to local disturbances in the neighborhood of the body and may presumably be used for the calculation of the added mass quantities. This manner of separation, however, has not been accomplished as yet; on the other hand, a direct calculation of the applied forces leads in principle to the goal, although it has been actually successful only in a few cases. Independent of the steady or unsteady nature of the body motion, the force and moment vectors are given by

$$F = \int_A \int pndA \qquad M = -\int_A \int pr.ndA \qquad [4]$$

where n indicates the exterior normal and r the radius vector. The integration is performed over the surface of the body.

The pressure p is determined by the instantaneously existing flow field, the structure of the latter, however, is in general not only a function of the instantaneous velocity and acceleration but is also a function of the law according to which this acceleration dies out. In other words, the flow field and therefore the pressure and the forces developed depend on the history of the motion of the body.

This means that for a given speed and acceleration the added mass of a body may vary with the kind of the motion, for instance, assume different values for a translation, a free or a forced oscillation in the same direction.

Under these circumstances, the question, whether and to what extent the concept of hydrodynamic masses can still be maintained in the case of an accelerated motion of the body on a free surface, appears justified. We shall here anticipate the answer: the concept remains quite suitable, however, the quantities in question can be functions of various variables so that they lose their simple geometrical character.

Some simple formal reflections now follow.

2.2 If our body is moved on or in the neighborhood of the free surface, then our velocity potential ϕ must satisfy, in addition to Laplace's equation and the boundary condition on the body, the linearized boundary condition of the free surface

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} = 0 \quad \text{for } z = 0 \qquad [5]$$

We shall consider two important special cases:

a. The ship undergoes a uniform translational motion U; then one writes in place of [5]

$$\frac{\partial^2\phi}{\partial x^2} + \frac{g}{U^2}\frac{\partial\phi}{\partial z} = 0 \quad \text{for } z = 0 \qquad [6]$$

b. The ship oscillates harmonically in position. If one sets

$$\Phi(x,y,z,t) = \Phi_0 \, e^{i\omega t}$$

one obtains

$$\Phi_0 - \frac{g}{\omega^2} \frac{\partial \Phi_0}{\partial z} = 0 \qquad [7]$$

Furthermore, if we introduce an appropriate length l and if we choose l equal to the ship length L in Case a so that

$$\phi = \phi_1 L, \qquad x = x_1 L$$

this immediately gives

$$\frac{\partial^2 \phi_1}{\partial x_1^2} + \frac{gL}{U^2} \frac{\partial \phi_1}{\partial z_1} = 0 \qquad [6a]$$

or

$$\frac{\partial^2 \phi_1}{\partial x_1^2} + \frac{1}{F^2} \frac{\partial \phi_1}{\partial z_1} = 0 \qquad [6b]$$

where $F = \frac{U}{\sqrt{gL}}$ is the usual Froude number.

In the oscillation problem, Case b, it is obvious that l should be set equal to the beam of the ship B, i.e., l = B. With $\Phi_0 = \Phi_1 B$, $z = z_1 B$, Equation [7] becomes

$$\Phi_1 - \frac{g}{B\omega^2} \frac{\partial \Phi_1}{\partial z_1} = 0 \qquad z_1 = 0 \qquad [7a]$$

or

$$\Phi_1 - \frac{1}{F_\omega^{\,2}} \frac{\partial \Phi_1}{\partial z_1} = 0 \qquad z_1 = 0 \qquad [7b]$$

Here the important dimensionless oscillation parameter $F_\omega = \omega\sqrt{B/g}$ is introduced.

We see that in both of the special cases considered the potential and therefore the hydrodynamic masses depend on the Froude number F and the oscillation parameter F_ω, respectively. These cases can be extended to the case of a uniform or accelerated translational motion with harmonic oscillation; for our purpose, however, Equations [6] and [7], which will now be discussed together, are sufficient. In this joint discussion of the two equations, we shall use the same symbol ϕ for both.

2.3 Let us consider two limiting cases:

a. The velocity or the frequency is extremely small, i.e., one can set $F^2 \to 0$ or $F_\omega^2 \to 0$. Equations [6] and [7] transform into the simple boundary condition

$$\frac{\partial \phi}{\partial z} = 0 \qquad z = 0 \tag{8}$$

i.e., the vertical velocities vanish or the surface acts as a solid boundary.

b. In the case of extremely high velocities or frequencies, i.e., $F^2 \to \infty$ or $F_\omega^2 \to \infty$, the boundary conditions simplify to

$$\frac{\partial^2 \phi}{\partial x^2} = 0 \quad \text{and} \quad \phi = 0 \qquad z = 0 \tag{9}$$

It can be shown, although with some difficulties, that the condition $\frac{\partial^2 \phi}{\partial x^2} = 0$ likewise leads to the condition $\phi = 0$.

Physically, this boundary condition indicates, as is well known, that we can neglect gravity in comparison to the inertia forces. Therefore, this is true in particular for impact phenomena on the surface.

The solution for the hydrodynamic masses obtained on the basis of the simplified boundary conditions $\frac{\partial \phi}{\partial z} = 0$ and $\phi = 0$ are self-evidently no longer dependent on the parameters F and F_ω, and consequently fail as general solutions of the problem. An indiscriminate application of the results, as frequently occurs at the present time, is therefore misleading. On the other hand, they represent important limiting cases and reveal some interesting facts so that it is worthwhile to discuss them in some detail.

First of all, we shall complete the system of our symbols. To differentiate from the previously introduced concepts $k_x, \ldots, m_x, \ldots$ (briefly called the "deeply immersed" values) and $\tilde{k}_x, \ldots, \tilde{m}_x, \ldots$ (the "wave" values), we shall designate the hydrodynamic mass values for the solid boundary $\frac{\partial \phi}{\partial z} = 0$ by $\bar{k}_x, \ldots, \bar{m}_x, \ldots$ and for the free surface neglecting gravity $\phi = 0$ by $\mathring{k}_x, \ldots, \mathring{m}_x, \ldots$.

We obtain a physical understanding of the relations between the values $\bar{k}_x$, $\mathring{k}_x$, etc., and the deeply immersed values k_x from the following sketches:

We mirror first the immersed part of the body S, which in principle may have an arbitrary shape, at the line OY. Thus above the axis a body S', symmetric to S, is generated.

In Figures 1 and 2 we consider a vertical translation of the body and indicate the resulting motion of the body by introducing sources and sinks. Clearly, the disturbances are larger for the rigid wall than for the

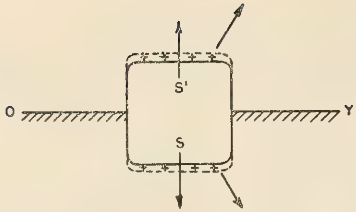

Figure 1 - Vertical Motion, Rigid Wall

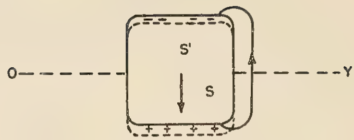

Figure 2 - Vertical Motion, Free Surface

free surface without gravity; we expect therefore that

$$\bar{k}_z > \mathring{k}_z \qquad [10]$$

One derives further from Figure 2 that

$$\mathring{k}_z = k_z \qquad [11]$$

since S + S' together behave like a double body. Hence, the coefficient $\mathring{k}_z$ is known when k_z is given. Similarly, as for Equation [11], we conclude

$$\mathring{k}_{yy} = k_{yy} \qquad [11a]$$

$$\mathring{k}_{xx} = k_{xx} \qquad [11b]$$

Figures 3 and 4 explain conditions when the body moves <u>horizontally</u>. The rigid wall (Figure 3) has the function of a plane of symmetry; S + S' move again together as a double model. We conclude therefore that in this case

$$\bar{k}_y = k_y \qquad [12]$$

and by analogy

$$\bar{k}_{zz} = k_{zz} \qquad [12a]$$

$$\bar{k}_x = k_x \qquad [12b]$$

Figure 4 pictures the conditions at the free surface; obviously, one follows that $\mathring{k}_y < \bar{k}_y$ and therefore $\mathring{k}_y < k_y$. The same applies to the other components.

The difference in the results obtained for motions in a vertical and a horizontal plane is essential. Using one of the approximate boundary conditions, we can substitute a double model for the ship only for a single direction of motion. The difference in the effects involved is appreciable: in the case of an elliptic cylinder which floats at its plane of symmetry, the ratio $\mathring{k}_y/k_y$ is only $4/\pi^2$.

$$\frac{\mathring{k}_y}{k_y} \doteq 0.4 \qquad [12c]$$

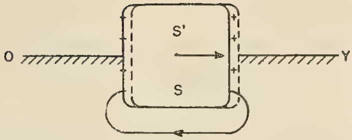

Figure 3 - Horizontal Motion,
Rigid Wall

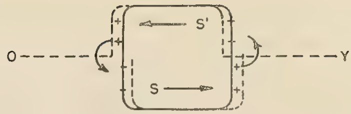

Figure 4 - Horizontal Motion,
Free Surface

To my knowledge the evaluation of hydrodynamic masses for the boundary condition $\phi = 0$ has been carried out comprehensively for the first time by Lockwood Taylor.[5]

The foregoing reasoning can be illustrated by well known first order results for the circular cylinder and the sphere (Figure 5). The ratio radius r, over depth of immersion f, is assumed to be small.[1]

For a motion in a horizontal plane Z = -f parallel to the wall and to the free surface one obtains:

Cylinder

$$\bar{k}_y = 1 + \frac{1}{2}\left(\frac{r}{f}\right)^2 \qquad [13]$$

$$\mathring{k}_y = 1 - \frac{1}{2}\left(\frac{r}{f}\right)^2 \qquad [13a]$$

Sphere

$$\bar{k}_y = 1 + \frac{3}{16}\left(\frac{r}{f}\right)^3 \qquad [14]$$

$$\mathring{k}_y = 1 - \frac{3}{16}\left(\frac{r}{f}\right)^3 \qquad [14a]$$

In the case of a motion vertical to the wall (surface) similar expressions with larger correction terms are valid.[1]

As compared with the inertia coefficients calculated for infinite depth of immersion, $f \to \infty$, a dependence upon the wall distance arises. It is easy to estimate the magnitude of f for which this wall influence disappears. Such estimates also will be valuable for the general case of a fluid with gravity; this can be, for instance, important in experimental tank work.

The change in the sign of the correction term [13] and [13a] for the wall and the free surface is due to the difference in the procedure of mirroring the images involved.

One could assume that the "wave values" $\tilde{k}$ are included between $\bar{k}$ and $\mathring{k}$ as bounds. This may be frequently a useful approximation, but is by no means exact, as will be proved below.

It will be shown later that the boundary condition $\phi = 0$ leads to a further important solution of the general case.

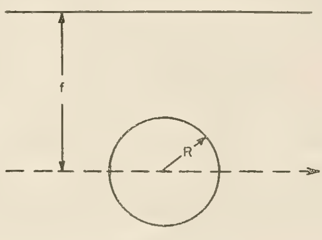

Figure 5

In general, however, we must look for a better mechanical model than that presented by the rigid wall or the free surface neglecting gravity.

2.4 It is natural to substitute systems of singularities (sources or dipols) for the body (ship) and to calculate the hydrodynamic masses using these images and the boundary condition, Equation [5]. Following a procedure known in the theory of wave resistance, these images apparently could be determined using a uniform flow in an unbounded medium. Since this approach is successful when calculating the resistance at uniform speeds of translation, it has been applied several times to investigations of accelerated translations and oscillations in calm water and in a seaway.[6,7] The result is surprising; the hydrodynamic inertia factors like $\tilde{k}_z$ $\tilde{k}_x$ found in such a way become equal to $\mathring{k}_z$ $\mathring{k}_x$, etc.; there is no dependence upon the Froude number or other characteristic parameters.

This obvious contradiction to simple physical reasoning has not been detected for some time. It induced the author to make some similitude investigations; from these the importance of such parameters as F, F_ω, and a/g and the history of the flow field has been realized when calculating the added masses. In the meantime, three fundamental papers have been published which clarified, at least in principle, the intricate problem, although an exhaustive solution is still lacking.

2.5 For the special case of a horizontal accelerated motion of a circular cylinder, Sir T.H. Havelock proved that the hydrodynamic masses can only be found by using a second approximation for the image distribution.[8]

He further obtained explicit values when the acceleration of the cylinder a is constant. Beside the depth of immersion ratio r/f discussed before, the ratio a/g becomes a decisive parameter. An interesting diagram[8] shows the coefficient $\tilde{k}_y$ as function of the Froude number F when the cylinder has been accelerated from rest with a given a/g = constant. It is surprising that $\tilde{k}_y = \mathring{k}_y$ holds not only for $F \rightarrow \infty$, which is well known, but also for $F \rightarrow 0$. Otherwise expressed, in the present case of a uniform acceleration, the hydrodynamic masses in the starting condition are the same as for an impulsive motion; the influence of gravity can be neglected. Starting conditions are especially important in the theory of directional stability.

Unfortunately, at present we are not yet able to generalize the results obtained under the assumption a/g = constant for other acceleration laws which may be more realistic. But following Havelock, another generalization seems to be plausible: the result $\tilde{k}_y = \mathring{k}_y$ remains valid for the start of the constantly accelerated motion when the acceleration begins not at U = 0, but even at $U \neq 0$.

From Havelock's investigation, the fact, mentioned before, follows that $\bar{k}_y$ and $\mathring{k}_y$ are not bounds of $\tilde{k}_y$.

There exist two pertinent publications on forced heaving oscillations in the moored condition. They deal with the dependence of the hydrodynamic masses upon the frequency of the oscillation. Figure 6 shows results for a circular cylinder immersed up to a plane of symmetry following F. Ursell[9] and for a ship model following M. Haskind.[10] Remarkable is the hump in the $\tilde{k}_z$ curve in the range of small frequency parameters $F_\omega = \omega\sqrt{B/g}$; it indicates that the condition $\partial\phi/\partial z$ is approximately satisfied. We emphasize further that in a certain region $\tilde{k}_z$ becomes smaller than $\mathring{k}_z = 1$, which again means that the condition $\phi = 0$ does not furnish a bound for $\tilde{k}_z$. Unfortunately, nothing is known about the method used by Haskind; we stress, however, the importance of the curve of experimental values due to the same distinguished scientist. We face now the question as to how far experiments in general have contributed to our knowledge in this field.

2.6 When the derivation of theoretical solutions is difficult, the experimental approach suggests itself. In the field of wave resistance of ships, an overwhelming number of mostly unsystematic tests have been performed which nevertheless have yielded important results. Since the problem of added masses is less important from a practical viewpoint, the amount of experimental work is small and correspondingly the success too is modest. The last fact becomes especially evident because in older publications even the decisive parameters of the problem remained unknown.

Experiments dealing with added masses of ships in accelerated and retarded translations (inertia tests) are especially precarious. The longitudinal coefficient $\tilde{k}_x$ is normally small; additionally we must consider that the wave resistance in its proper sense, and even the frictional resistance, depend upon the acceleration. In the light of our present knowledge, the foundation of inertia tests which in principle are very suggestive, must be reconsidered. The determination of $\tilde{k}_x$ by such experiments is not promising, at least for the time being; on the contrary, one would prefer to estimate $\tilde{k}_x$ and derive thus conclusions about the magnitude of the resistance.

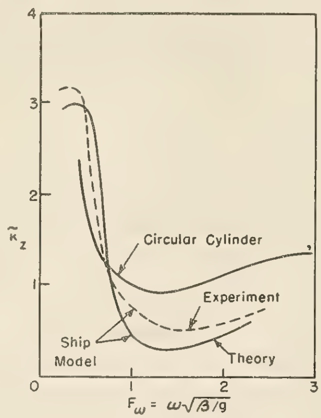

Figure 6

Let us finally consider oscillations. Roll experiments are a well-established requisite of the testing technique. However, the author does not know of any systematic experiment dealing with $\tilde{k}_{xx}$ as a function of the speed of advance or the frequency of roll. It has been reported without further explanations that a variability of the period with the Froude number has been observed on a model of the Conte di Savoia.[11] Probably it would be worthwhile to check the few pertinent publications if a dependence of $\tilde{k}_{xx}$ upon F can be stated; obviously, possible variations of the metacentric height must be known.

Some twenty years ago I recorded extinction curves of the heave and pitch motion; the work was carried out at the Berlin Model Basin.[12] We investigated a full and a moderately-full model. Within the range of Froude numbers $0 \leq F \leq 0.20$ one could not find a dependency of the inertia coefficients $\tilde{k}_z$, $\tilde{k}_{yy}$ and of the damping $2\delta_z$, $2\delta_{yy}$ upon the speed. The measured values of $\tilde{k}_z$, $\tilde{k}_{yy}$ were slightly less than those computed by the "strip method with ellipsoidal corrections" (see Section 1.4). Later, similar experimental results for ship models have been published only occasionally.

More detailed heaving experiments with geometrically defined bodies have been performed by Dimpker[13] and Holstein.[14] Although Holstein's experiments were carefully conducted, their accuracy is not high enough to establish a consistent dependency of the added mass values upon F_ω. Following Wendel, the experimental values of k_z plotted over the beam-draft ratio are somewhat lower than the corresponding "theoretical" values of k_z. Holstein was unable to detect the hump in the range of small F_ω since the frequencies tested correspond to a range $F_\omega > 1.5$.

As has been mentioned before, the usefulness of the added mass concept can be questioned when its value becomes a function of many variables. However, it can be asserted that the concept remains important since

1. With increasing depth of immersion f the values $\tilde{k}$ approach quickly the "classical" values k.

2. The order of magnitude of $\tilde{k}$ is generally identical with that of k.

3. The solutions $\bar{k}$, $\mathring{k}$, and k obtained under simplified conditions, yield over large ranges, valuable estimates for $\tilde{k}$.

We mention two special problems, shallow water effects and gravity effects. In a well-known paper I.I. Koch has calculated the inertia factors k_x and k_y of a rectangle, using an electrical analogy.[15] Schnadel has successfully applied these results to the interpretation of vibration experiments.[16] Since gravity effects so far have not been considered, we restrict

ourselves to this remark.

Finally, the influence of viscosity must be mentioned. Except for small Reynolds numbers, no information is available on the subject. One makes the plausible assumption that because of the small displacement thickness of the boundary layer, the influence of viscosity on the magnitude of the added masses is negligible.

3. SUMMARY

The hydrodynamic masses of a body moving at the free surface are mathematically formulated functionals of the body motion. In principle, beside the form of the body, the history of its motion and thus the resulting velocity field must be given.

Only few solutions for some simple kinds of motion are known, from which the dependence of the added masses from an acceleration parameter a/g, the Froude number F, a frequency parameter F_ω, etc., can be established. From a physical point of view we are just beginning to tackle the problem.

From a point of view of technical application, conditions are slightly more favorable, since one is able to make estimates which, however, frequently are rather coarse. The most familiar estimates are:

For oscillations in a vertical plane, one assumes

$$\tilde{k}_z \doteq k_z, \quad \tilde{k}_{yy} \doteq k_{yy}$$

The values $\tilde{k}_z$, $\tilde{k}_{yy}$ are, at least up to moderate Froude numbers, rather independent of the speed of advance. Because of the high values of the frequency parameter involved, one can hope that the mentioned approximate relations may even be accurate enough for calculating free vibration periods.

But, again, we do not possess a satisfactory general solution and serious endeavors will be needed in the field of theory and experiment to improve the present state of knowledge.

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